

Extended King Models for Star Clusters

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Abstract

Models applied to galaxies in equilibrium configuration are based on the solution of the collisionless Boltzmann equation and the Poisson equation for gravitational interaction which are related to each other by a smoothed-out mass density. This mass density is related to a distribution function $f(\mathbf{x}, \mathbf{v}, \mathbf{t})$ and produces a gravitational potential $\phi(\mathbf{x}, \mathbf{t})$. The most popular models to describe spherically symmetric systems are King models. These models are based on a truncated isothermal sphere. They fit the surface brightness of globular clusters and some elliptical galaxies well. A well-known example of a galaxy that King models does not fit well is NGC 3379. In this work, we extend King models. Our models are based on the Tsallis distribution function. There is a parameter q in this function and we can recover the King distribution function in the limit $q \rightarrow 1$. We discuss the general behaviors of our models and, moreover, we use them to fit the surface brightness of NGC 3379 and 47 Tuc.

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A lot of models for star clusters, based on the collisionless Boltzmann equation and Poisson equation, have been studied in the last century [1,2]. In this approach, the state of the system is described by a distribution function $f(\mathbf{x}, \mathbf{v}, \mathbf{t})$ in phase space and considering the smooth gravitational potential. These models can be applied to star clusters in equilibrium configuration or when the characteristic time of these structures does not exceed the relaxation time [1].

The most commonly used models to describe spherically symmetric systems are King models. These models can fit the brightness distributions of globular clusters and some elliptical galaxies well [1–5]. King models are based on the following distribution function

$$f_K(\varepsilon) = \begin{cases} \rho_1 (2\pi\sigma^2)^{-3/2} (e^{\varepsilon/\sigma^2} - 1) & \varepsilon > 0 \\ 0 & \varepsilon \leq 0 \end{cases} . \quad (1)$$

where ε is the relative energy $\varepsilon = -E + \Phi_0 = \Psi - v^2/2$ and Ψ is the relative potential $\Psi = -\Phi + \Phi_0$. This distribution function may be viewed as a truncated isothermal sphere. The characteristic of this model is that the relative potential Ψ and density of stars ρ become equal to zero at some finite radius r_t (tidal radius). The value of r_t depends on the central potential $\Psi(0)/\sigma^2$ and King radius defined as $r_0 = \sqrt{9\sigma^2/4\pi G\rho_0}$. In the limit $\Psi(0)/\sigma^2 \rightarrow \infty$, the isothermal sphere is recovered and r_t it extends to infinity.

In this work, we generalize King models. The main goal of this generalization is to try to describe some elliptical galaxies where King models cannot describe them well. Thus, we are also restricted to spherically symmetric models and the distribution function depends only on relative energy $f(\varepsilon)$. These kinds of models possess many solutions for the collisionless Boltzmann equation: Jeans stated that any steady-state solution can be expressed through integrals of motion on phase-space coordinates. Moreover, all these models are stable [1].

Our distribution function is based on Tsallis' statistics [6], with the entropy and distribution function given by $S = k(1 - \sum_i p_i^q)/(1 - q)$ and $f_T(\varepsilon) = k(1 + \beta(1 - q)\varepsilon)^{1/(1-q)}$, respectively. In this type of statistics, q represents a parameter which describes different statistics. It has been successfully applied to several systems such as Lévi-type anomalous superdiffusion [7], Euler turbulence [8], and anomalous relaxation electron-phonon interaction [9]. In astrophysics and cosmology we can mention the following set of works [10]; but for our purposes, it is worthwhile to mention the work of Plastino and Plastino [10]. They have shown that Tsallis' entropy can determine the meaningful distribution function as described by polytropic models. Therefore, our distribution function, based on Tsallis' distribution function, is chosen as follows:

$$f(\varepsilon) = \begin{cases} \rho_1 (2\pi\sigma^2)^{-3/2} \left\{ \left[1 + (1 - q)\frac{\varepsilon}{\sigma^2} \right]^{\frac{1}{1-q}} - 1 \right\} & \varepsilon > 0 \\ 0 & \varepsilon \leq 0 \end{cases} , \quad (2)$$

where q is a real parameter. This distribution function incorporates King's distribution function and recovers it in the limit $q \rightarrow 1$. For simplicity, we also assume that the stars all have the same mass and the velocity distribution is isotropic everywhere. The dependence on r is determined by Poisson equation

$$\nabla^2 \Psi = -4\pi G\rho , \quad (3)$$

where ρ is the density given by

$$\rho = 4\pi \int_0^{\sqrt{2\Psi}} f\left(\Psi - \frac{1}{2}v^2\right) v^2 dv. \quad (4)$$

Substituting Eq. (4) into (3) we obtain

$$\frac{d}{dr} \left(r^2 \frac{d\Psi}{dr} \right) = - \frac{(4\pi)^2 G \rho_1 r^2}{(2\pi\sigma^2)^{3/2}} \int_0^{\sqrt{2\Psi}} \left\{ \left[1 + (1-q) \frac{\varepsilon}{\sigma^2} \right]^{\frac{1}{1-q}} - 1 \right\} v^2 dv. \quad (5)$$

For convenience, we put the last equation in terms of Ψ/σ^2 and dimensionless quantity $\bar{r} \equiv r/r_0$, and it becomes

$$\frac{d}{d\bar{r}} \left(\bar{r}^2 \frac{d}{d\bar{r}} \left(\frac{\Psi}{\sigma^2} \right) \right) = - \frac{9\bar{r}^2 \left\{ \int_0^{\frac{\sqrt{\Psi}}{\sigma}} \left[1 + (1-q) \left(\frac{\Psi}{\sigma^2} - u^2 \right) \right]^{\frac{1}{1-q}} u^2 du - \frac{1}{3} \left(\frac{\Psi}{\sigma^2} \right)^{\frac{3}{2}} \right\}}{\int_0^{\frac{\sqrt{\Psi(0)}}{\sigma}} \left[1 + (1-q) \left(\frac{\Psi(0)}{\sigma^2} - u^2 \right) \right]^{\frac{1}{1-q}} u^2 du - \frac{1}{3} \left(\frac{\Psi(0)}{\sigma^2} \right)^{\frac{3}{2}}}. \quad (6)$$

This equation should be integrated numerically. We use the Runge-Kutta method to calculate it. As usual, we choose the initial condition for $d\Psi/dr = 0$ at $r = 0$. Different models can be obtained by giving the central potential $\Psi(0)/\sigma^2$ and the value of q . For $q > 1$, the validity of Eq. (6) is restricted by the condition $(q-1) < 1/[\Psi(0)/\sigma^2]$ in order to maintain the quantity inside of the square brackets positive.

In order to compare the theory with observational data, the density calculated from Eqs. (4) and (6) is substituted into the projected density $\Sigma(r)$ formula that is given by

$$\frac{\Sigma(r)}{\rho_0 r_0} = 2 \int_{\bar{r}}^{\infty} \frac{\bar{\rho}(\tilde{r}) d\tilde{r}}{\left[1 - \frac{\bar{r}^2}{\tilde{r}^2} \right]^{\frac{1}{2}}}, \quad (7)$$

where $\bar{\rho}(\tilde{r}) = \rho(\tilde{r})/\rho_0$.

We should point out that King models are parameterized by the central potential $\Psi(0)/\sigma^2$ or the concentration $c = \log_{10}(r_t/r_0)$. In these models, the tidal radius or c is uniquely determined by the value of the central potential. The greater the value of the central potential, the greater the tidal radius. In our models, the concentration is not uniquely determined by the central potential, but we can adjust the additional parameter q . In Fig.1 we show the density profile for $\Psi(0)/\sigma^2 = 12$ with different values of q . We can see that the tidal radius increases with the value of q . In Fig.2 we show the surface brightness $I(r)$ of our models, on the scale $\mu = -2.5 \log I(r)$ versus $r^{1/4}$, with different values of q . We also note that the curves become shallow with the decrease in the value of q , i.e., for $q > 1$, the curves become more crooked than those of $q = 1$, and for $q < 1$, the curves become more straight in some range and they become more and more straight with the decrease in the value of q . In the latter case, the larger the value of $\Psi(0)/\sigma^2$, the longer the straightend part of the curve becomes. This fact is important to the fitting of some elliptical galaxies, (for example NGC3379) because the surface brightness of King models cannot run straight over 10 magnitudes with the increase in tidal radius and their curves become more crooked (see Fig. 2). In Fig.3, we fit the surface brightness of galaxy NGC3379. The value of r_0 is chosen so that the theoretical curve falls on the observational data. We see that our theoretical

curve can fit the data of the outer region and runs almost straight over 10 magnitudes, in contrast to the King model which runs less than 10 magnitudes. In Fig.4 we fit the surface brightness of globular cluster (47 TUC). We note that the King model ($c=2.03$) fits tolerably well, in contrast with our model which fits very well.

In summary, we have extended King models to describe spherically symmetric systems. Our models are based on the distribution function (2) and can be viewed as a truncated Tsallis distribution function. In these models, we have considered that the stars all have the same mass and the velocity distribution is isotropic everywhere. We have analyzed the general properties of these models and contrasted them with King models ($q = 1$). As applications of our models, we have fitted the surface brightness of NGC3379 and 47 TUC. From Fig.3, we see that the model with $q \neq 1$ fits the surface brightness of the galaxy tolerably well, in contrast to the case of $q = 1$ which cannot fit the outer region. Moreover, the improvement of our model in relation to the King model is clear from Fig.4. These results suggest that the velocity distribution of stellar systems described here deviates from Maxwellian shape. Of course, our models represent the simplest of models, but can be extended to include the different masses and anisotropy of actual clusters.

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FIGURE CAPTIONS

Fig.1 - Density profile plots with $\Psi(0)/\sigma^2 = 12$ and different values of q .

Fig.2 - Surface brightness plots versus $r^{1/4}$ with $\Psi(0)/\sigma^2 = 8, 12$ and different values of q . The dotted curves correspond to $\Psi(0)/\sigma^2 = 8$: from bottom to top the value of q is given by $q = 0.92, 0.96, 1, 1.02, 1.04$. While, the full curves correspond to $\Psi(0)/\sigma^2 = 12$: from bottom to top the value of q is given by $q = 0.6, 0.8, 1, 1.02, 1.04$.

Fig.3 - Surface brightness fit of galaxy NGC3379 measured by different observers along the galaxy's east-west axis (from data published by Vaucouleurs and Capaccioli [4]). The dashed curve corresponds to $\Psi(0)/\sigma^2 = 9.2$ and $q = 1$, whereas the full curve corresponds to $\Psi(0)/\sigma^2 = 12.2$ and $q = 0.939$.

Fig.4 - Surface brightness fit of globular cluster 47 TUC by different observers (from data published by Illingworth & Illingworth [4] and King et. al., 1968 [4]). The dotted curve corresponds to $\Psi(0)/\sigma^2 = 8.868$ and $q = 1$, whereas the full curve corresponds to $\Psi(0)/\sigma^2 = 10$ and $q = 0.966$.